

Mar 5 Bamler - Cifarelli - Condon - Deruelle

BCCD soliton

Zhu (+ [CCD 22]) classifies Kähler gradient shrinkers  
in complex 2D. complete, bdd scalar

Zhu (Li-Wang) complete grad shrinker Kähler  
→ bdd scalar.

i.e. such a shrinker is

compact (i) Kähler-Einstein on Fano ( $c_1(M) > 0$ )  
or compact shrinker Wang-Zhu.

noncompact  $\Rightarrow$  connected to  $\infty$ .

Kähler form  $\omega$ . locally  $i\partial\bar{\partial}\varphi$ ,  $\varphi \in C^\infty(M; \mathbb{R})$ .

Ricci form  $Rsc(\omega)(\cdot, \cdot) = Rsc(g)(J, \cdot)$   
 $= i\partial\bar{\partial} \log \det g$

Shrinking gradient Kähler Ricci soliton

$(M^{2n}, g, J, \nabla f)$  holomorphic  
 $Rsc(g) + \nabla^2 f = g$ .

- $R(g) \rightarrow 0$  along all integral curves of  $\nabla f$
- (ii)  $\mathbb{C}^2$  flat Gaussian shrinker  $\nabla f = r \partial r$ .
- (iii) "blowdown" / FZK shrinker.  
 $U(2)$  invariant on  $Bl_0(\mathbb{C}^2)$ .

- $R_{ij} \rightarrow 0$  along some  $\nabla$  trajectory:  $(iv) \mathbb{C} \times \mathbb{P}^1$
- (v) [CCD22] a toric Kähler shrinker on  $\mathbb{P}^1 \times (\mathbb{C} \times \mathbb{P}^1)$   
(if it exists, unique)  $\rightarrow$  (Thm A)

$$\partial_t R = \Delta R + 2|Ric|^2 \geq \frac{2}{n} R^2 + \Delta R. \quad \left. \begin{array}{l} R \geq 0 \text{ for} \\ \text{shrinker.} \end{array} \right\}$$

$$R \sim \frac{1}{t} \quad (-\infty, 0)$$

Kähler-Ricci flow  $(M^m, \omega, J)$

$$\partial_t \omega = -Ric(\omega)$$

$(M^m, \omega(t))$  remains Kähler w.r.t the fixed  $J$ .

since  $c_1(M) \cdot \pi = [-Ric(\omega)]$

$$\Rightarrow [\omega(t)] = [\omega_0] - \pi c_1(M) t \in \text{"Kähler cone"}$$

$$C_M = \{[\omega] \mid \text{admits Kähler form w.r.t. } J\} \rightarrow$$

Thm if  $[\omega_0] - \pi c_1(M) t \in C_M$  then KRF exists up to  $t$ .

Finite time singularities.  $T < \infty$  on  $[0, T)$ .

- $\sup (T-t) |R_{i\bar{j}}| < \infty$  Type I (\*)
- $\sup (T-t) |R_{i\bar{j}}| = \infty$  Type II

Enders-Morlier-Topping (\*)  $\rightarrow$  rescale  $\rightarrow$  grad. shrinker.

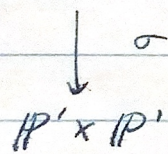
Outline of the pf of Thm A  $= N$ .

1. toric KRF on  $Bl_x(\mathbb{P}^1 \times \mathbb{P}^1)$  that contract ~~exp~~ exceptional divisor at  $T_{\text{cus}}$  and volume control.
2. Bamler, analyze blowup, exclude orbifold sing. Show the sing is Type I.
3.  $\text{vol}(N) \rightarrow 0$  as  $t \rightarrow T$ . rule out FIK by  $[CED^2] \Rightarrow (N) \text{ or } (V)$ .  
volume computation  $\rightarrow$  not  $\mathbb{C} \times \mathbb{P}^1$ .

1. metric metric  $S' \times S'$  on  $\mathbb{P}^1 \times \mathbb{P}^1$ .

p.g fixed point of  $S' \subset \mathbb{P}^1$ , let  $x = (p, \theta)$ .

Take  $Bl_x(\mathbb{P}^1 \times \mathbb{P}^1) = N$



$E$  exceptional divisor.

$$D_1 = \sigma^{-1}(\mathbb{P}^1 \times \{q\}), \quad D_2 = \sigma^{-1}(\{\emptyset\} \times \mathbb{P}^1).$$

$$\text{Intersection form } [D_1]^2 = 0, \quad [D_1][D_2] = 1$$

$$[D_1][E] = 0, \quad [E]^2 = -1$$

Lemma - can choose  $g(t)$ , Kähler on  $N$  st.

$$\text{vol}(D_1, g(t)) \rightarrow 0$$

$$\text{vol}(D_2, g(t)) \rightarrow 0$$

$E$

$$\text{vol}(N, g(t)) (T-t)^{-2} \rightarrow 0$$

$N$

2. by contradiction. if not  $\exists (x_i, t_i)$  st  
 $|Rm(x_i)| (T - t_i) \rightarrow \infty$

Bamler's compactness theory  $\Rightarrow$

$(N, (T - t_i)^{-1} g((T - t_i, t + t_i), \forall x_i, t) \xrightarrow{\mathbb{F}\text{-conv}} (X, (V_\infty, t)_{t \leq 0})$

ancient metric flow  $X_\infty$  pt.

smooth away from codim  $\neq$  set and singular at  $X_\infty$

$\rightsquigarrow$  isolated orbifold sing.  $\mathbb{R}^4/\Gamma$ .  
 non flat, flow of metric soliton

$(N, g(t))$  type I sing as  $t \rightarrow T$ .

not compact

$R > 0$

vol  $> 0$  so not FZ/K

[CCD 22]

$\Rightarrow \mathbb{C} \times \mathbb{P}^1$  or  $(v)$ .