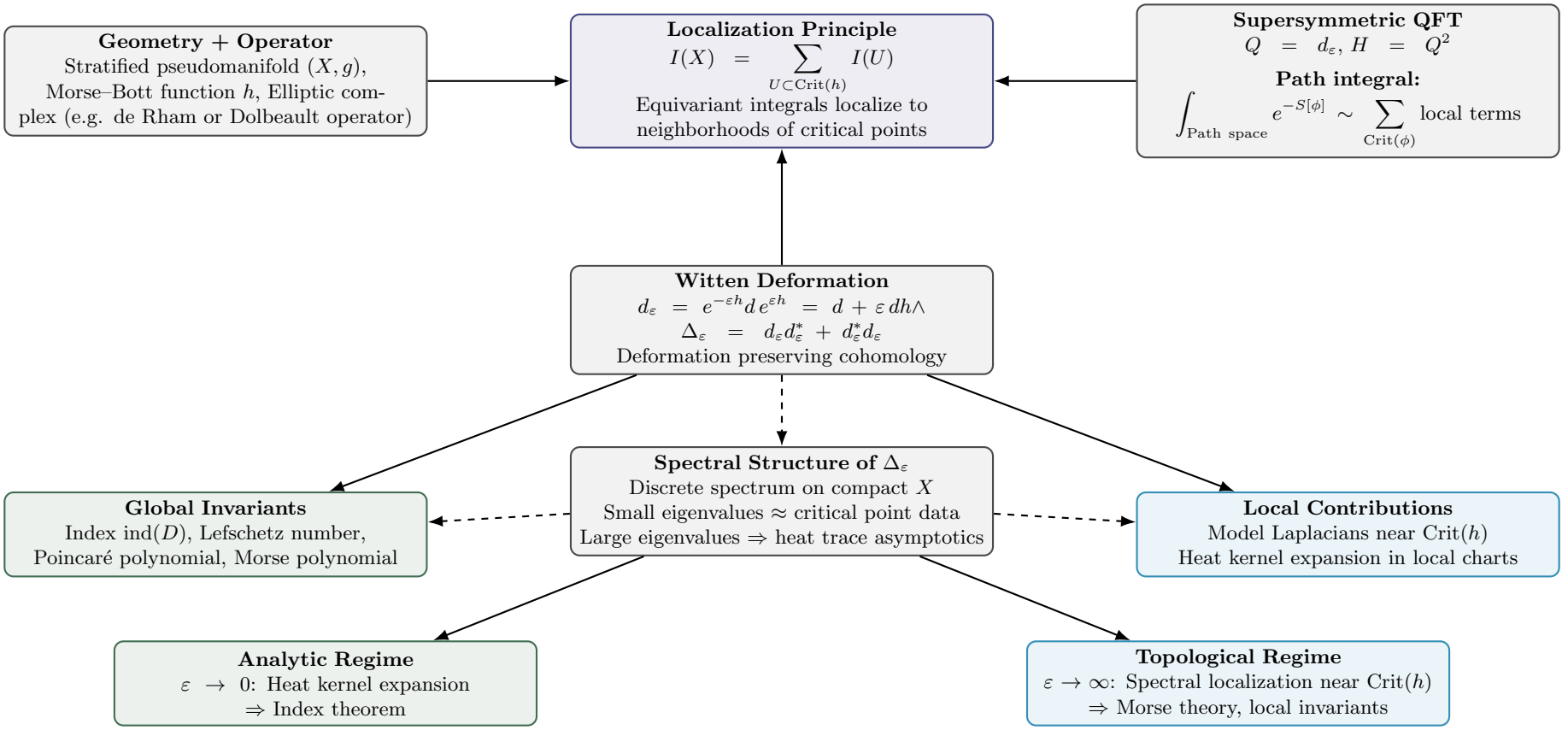
Geometry + Operator
Stratified pseudomanifold (X, g),
Morse–Bott function h, Elliptic complex (e.g. de Rham or Dolbeault operator)
Localization Principle
$I(X) = \sum_{U \subset \mathrm{Crit}(h)} I(U)$
Equivariant integrals localize to neighborhoods of critical points
Supersymmetric QFT
$Q = d_\varepsilon$, $H = Q^2$
Path integral:
$\int_{\text{Path space}} e^{-S[\phi]} \sim \sum_{\mathrm{Crit}(\phi)}$ local terms
Witten Deformation
$d_\varepsilon = e^{-\varepsilon h} d\, e^{\varepsilon h} = d + \varepsilon\, dh\wedge$
$\Delta_\varepsilon = d_\varepsilon d_\varepsilon^* + d_\varepsilon^* d_\varepsilon$
Deformation preserving cohomology
Spectral Structure of Δ_ε
Discrete spectrum on compact X
Small eigenvalues ≈ critical point data
Large eigenvalues ⇒ heat trace asymptotics
Global Invariants
Index ind(D), Lefschetz number,
Poincaré polynomial, Morse polynomial
Local Contributions
Model Laplacians near Crit(h)
Heat kernel expansion in local charts
Analytic Regime
$\varepsilon \to 0$: Heat kernel expansion
⇒ Index theorem
Topological Regime
$\varepsilon \to \infty$: Spectral localization near Crit(h)
⇒ Morse theory, local invariants