Historical Mathematical Models

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1. Project Goals

- Write **mathematical descriptions** about selected models for the University Library's Digital Library Collection.
- Use **Mathematica** and **Autodesk Fusion 360** to create digital visualizations and 3D-printable models for hands-on outreach activities.
- Develop **outreach materials** materials based on the selected models for use in local schools, science fairs, and public engagement events.

2. History

The University of Illinois owns one of the **largest collections** of historical mathematical models worldwide, comprising approximately **400 pieces** from the **late 19th and early 20th centuries**. These include models purchased from Germany and those created by faculty and students in the Department of Mathematics.

Edgar Townsend, the second chair of the department (1905–1928) and dean of the College of Science (1905-1913), played a pivotal role in acquiring many of these models, which feature works by the renowned German mathematicians **Alexander Brill**, **Martin Schilling**, and **Felix Klein**. **Arnold Emch**, hired by Townsend, later enriched the collection with many unique contributions [6]. After the ongoing renovations, the models will be re-displayed in Altgeld Hall, preserving their historical and educational value [5].

3. Schilling Series 40: Surfaces of Constant Width

There are three different three-dimensional models in this series. All of them are surfaces of constant width.

A **surface of constant width** is a convex body whose diameter—measured as the distance between parallel supporting planes in any direction—remains constant. As a result, such surfaces can "roll" smoothly between two parallel planes while maintaining continuous contact.

Here are the models from Schilling Series 40, from left to right:

- A rotationally symmetric algebraic surface of constant width
- A surface of revolution based on a Reuleaux triangle
- A Meissner tetrahedron, a surface of constant width

Figure 2: Surfaces of Constant Width in Series 40 [7]

4. Schilling Series 16: Confocal Quadrics

Each model is based on one of three **quadric surfaces**: an **ellipsoid**, a **hyperboloid of one sheet**, and a **hyperboloid of two sheets**. These models were made to showcase the unique intersections of confocal quadrics and how their intersections give a grid of orthogonal lines of curvature. Furthermore, these surfaces are defined by the equation

$$\frac{x^2}{a^2-\lambda}+\frac{y^2}{b^2-\lambda}+\frac{z^2}{c^2-\lambda}=1,$$

where

- $\lambda < c^2$ yields an ellipsoid,
- $c^2 < \lambda < b^2$ yields a hyperboloid of one sheet,
- $b^2 < \lambda < a^2$ yields a hyperboloid of two sheets.

The specific values of a, b, c, and λ for these models are derived from an 1888 paper by E. R. Neovius [4].

Figure 4: Three confocal quadrics

5. Schilling Series 32: Dandelin's Sphere

Dandelin's Theorem. Let P, P_1, and P_2 lie on a cone intersected by a plane, and let two inscribed spheres be tangent to the plane at points F_1 and F_2.

Then:

- $PP_1 = PF_1$ and $PP_2 = PF_2$.
- Hence, $PF_1 + PF_2 = P_1P_2$.

Thus, F_1 and F_2 are the foci, and the trajectory of P is an **ellipse**.

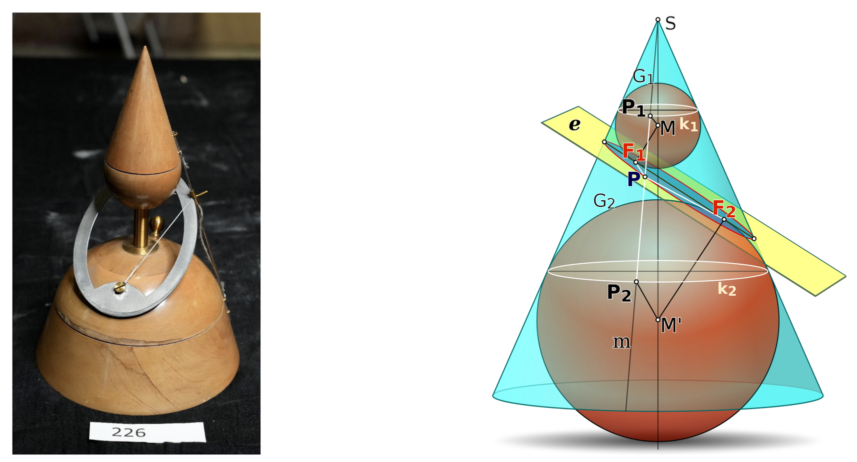

Figure 6: Dandelin spheres Model 2 Series 32 [7]

6. 3D Printing

Digitized various mathematical models for 3D printing to make them more accessible.

Originally designed to provide hands-on representations of mathematical concepts, these models can now reach a wider audience through digital modeling and 3D printing.

Software: Wolfram Mathematica, Autodesk Fusion 360

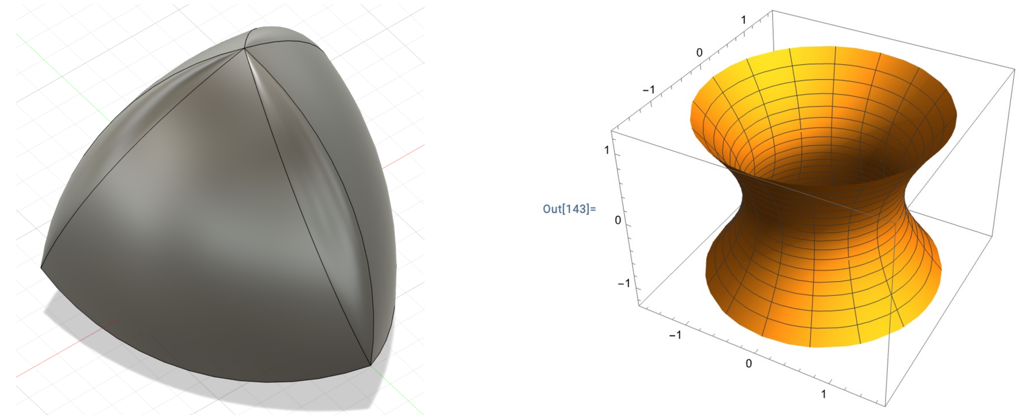

Figure 8: Examples of models in Fusion 360 and Mathematica, respectively

7. Outreach

Geometry is difficult for many students, and it is best to physically interact with many of the surfaces encountered by students. The purpose isn't just to get students excited about mathematics, but to also demystify and make this section of math more accessible and fun.

- Created a lesson plan centered around models and curves of constant width
- Main goal is to make geometry **interactive** and build a **"physical" intuition** for high dimensional surfaces
- Materials include (but are not limited to): **live simulation, arts and crafts, interaction with the models**

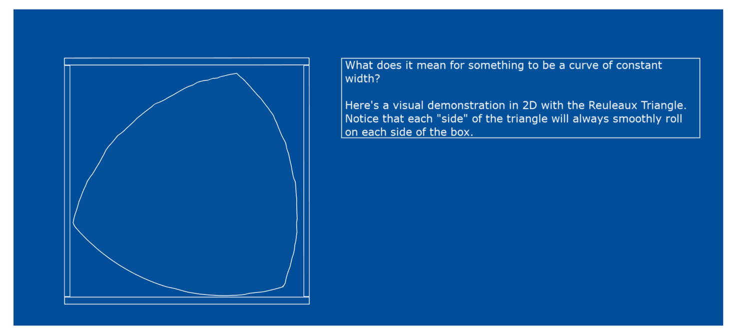

Figure 10: Making a simulation of the constant curvature of a Reuleaux Triangle + Video

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