Contents

MATH231 Spring 2022 Midterm 1 (50 points)

- **No notes, calculators, or other aids are allowed.**
- Read instructions carefully and write your answers in the space provided.
- There are 24 questions in total. You have 45 minutes to answer them.
- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify the formulae which haven't been given in class or derived in homework**.)
- Derivatives of inverse trigonometric functions

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\arctan x)' = \frac{1}{1+x^2}$$

Q1–Q4. Evaluate the following integrals. You may use any method other than the hint.

1. (8 points) $\int \arctan x \, \mathrm{d}x$

2. (8 points) $\int \frac{1}{\sqrt{x^2-2x}} \, \mathrm{d}x$ (Hint: trig substitution)

3. (8 points) $\displaystyle\int \frac{5x}{(x-2)(x+3)}\,\mathrm{d}x$

4. (10 points) $\displaystyle\int 16\sin^2 x\cos^4 x\,\mathrm{d}x$ (Hint: both powers are even)

Q5–Q6. Improper integrals.

5. (8 points) Use the definition to show that $\displaystyle\int_e^\infty \frac{1}{x\sqrt{\ln x}}\,\mathrm{d}x$ diverges.

6. (8 points) Use the comparison test to show $\displaystyle\int_\pi^\infty \frac{x\sin^2 x + 1}{x^4}\,\mathrm{d}x$ converges.

MATH231 Spring 2022 Midterm 2 (50 points)

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- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify any formula which haven't been given in class or derived in homework**.)
- You may use a different method other than the one asked in the question. However, the maximum points you can get will be half of the total points.

Integrals $I = \int f(u)\,\mathrm{d}u$ containing square roots, where a is a positive number.

$f(u)$	I
$\sqrt{a^2-u^2}$	$\dfrac{u\sqrt{a^2-u^2}+a^2\arcsin(u/a)}{2}+C$
$\sqrt{u^2-a^2}$	$\dfrac{u\sqrt{u^2-a^2}-a^2\ln\left\lvert u+\sqrt{u^2-a^2}\right\rvert}{2}+C$
$\dfrac{1}{\sqrt{u^2-a^2}}$	$\ln\left\lvert u+\sqrt{u^2-a^2}\right\rvert+C$
$\sqrt{a^2+u^2}$	$\dfrac{u\sqrt{a^2+u^2}+a^2\ln\left\lvert u+\sqrt{a^2+u^2}\right\rvert}{2}+C$
$\dfrac{1}{\sqrt{a^2+u^2}}$	$\ln\left\lvert u+\sqrt{a^2+u^2}\right\rvert+C$

Q1–Q2. Arc length and the surface of revolution.

1. (10 points) Compute the arc length of the curve $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$ for $1 \leq x \leq e$.

2. (10 points) Using surface of revolution integral to compute the surface area obtained by rotating the curve $y = \sqrt{r^2 - x^2}$ about the x-axis, where $r > 0$, constant and $-r \leq x \leq r$.

Q3–Q6. Series

3. (6 points) Compute $\sum_{n=0}^{\infty} \sqrt{5}^{1-2n} 2^{n+2}$. (Careful: n start from 0.)

4. (8 points) Use the divergence test to determine if $\sum_{n=1}^{\infty} \left(1 + \frac{1}{2n}\right)^n$ converges or not.

5. (10 points) Use the integral test to determine if $\sum_{n=1}^{\infty} ne^{-n^2}$ converges or not.

6. (6 points) Use the comparison test to determine if $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+n}}$ converges or not.

MATH231 Spring 2022 Midterm 3 (50 points)

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- There are 24 questions in total. You have 45 minutes to answer them.
- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify any formula which haven't been given in class or derived in homework**.)
- You may use a different method other than the one asked in the question. However, the maximum points you can get will be half of the total points.

1. (8 points) Use the limit comparison test to determine if $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^4+3n-1}}$ converges or diverges.

Q2–Q4. Determine if the alternating series is absolutely convergent, conditionally convergent or diverges. You may use any method.

2. (8 points) $\sum_{n=1}^{\infty} \frac{(-3)^n \cdot n^2}{(2n)!}$

3. (6 points) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

4. (10 points) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$

5. (10 points) Determine the radius of convergence R and interval of convergence I for the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{(-4)^n\sqrt{n}}$.

6. (8 points) Write $f(x) = \ln(x+5)$ as a power series centered at 0.

MATH231 Spring 2022 Final Exam (100 points)

- **No notes, calculators, or other aids are allowed.**
- Read instructions carefully and write your answers in the space provided.
- There are 24 questions in total. You have 45 minutes to answer them.
- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify any formula which haven't been given in class or derived in homework**.)
- You may use a different method other than the one asked in the question. However, the maximum points you can get will be half of the total points.

1. (8 points) Use the limit comparison test to determine if $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^4+3n-1}}$ converges or diverges.

Q2–Q4. Determine if the alternating series is absolutely convergent, conditionally convergent or diverges. You may use any method.

2. (8 points) $\sum_{n=1}^{\infty} \frac{(-3)^n \cdot n^2}{(2n)!}$

3. (6 points) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

4. (10 points) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n+1}+\sqrt{n}}$

5. (10 points) Determine the radius of convergence R and interval of convergence I for the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{(-4)^n\sqrt{n}}$.

6. (8 points) Write $f(x) = \ln(x+5)$ as a power series centered at 0.