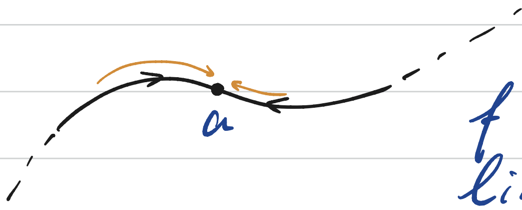
Review Calc I. (Limits Derivatives Integrals)

<u>Limit</u> intuitively



f

$$\lim_{x\to a} f(x) = f(a)$$

Calculating limits.

<u>Limit laws.</u>

1 $\lim\limits_{x\to c} f(x) \pm g(x) = \lim\limits_{x\to c} f(x) + \lim\limits_{x\to c} g(x)$

2 $\lim\limits_{x\to c} f(x)\, g(x) = \left(\lim\limits_{x\to c} f(x)\right) \lim\limits_{x\to c} g(x)$

3 $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$ when $\lim\limits_{x\to c} g(x) \neq 0$

why?

Example $\lim\limits_{x\to 0} \dfrac{(x+3)^2 - 9}{x}$ cannot apply 3 directly

Soln 1. direct computation

$$\lim_{x\to 0} \frac{x^2+6x+9-9}{x} = \lim_{x\to 0} \frac{x^2+6x}{x}$$

$$= \lim_{x\to 0} x+6$$

$$= \lim_{x\to 0} x + \lim_{x\to 0} 6 = 6$$

Soln 2. L'Hopital's rule

"$\frac{0}{0}$" "$\frac{\infty}{\infty}$"

Need to check $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ or ∞

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

$$= \lim_{x\to 0} \frac{2(x+3)}{1} = \lim_{x\to 0} 2x+6 = 6$$

Counter example to L'Hopital's rule

$$I = \lim_{x\to\infty} \frac{x}{x+\sin x}$$

$$-1 \le \sin x \le 1$$

$$\Rightarrow -\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

$$\downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0$$

$$\lim_{x\to\infty} \frac{1}{1+\frac{\sin x}{x}} = 1$$

Try to "apply L'Hoptial"

$$\lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \lim_{x\to\infty} \frac{1}{1+\cos x}$$

limit does not exist

Derivatives

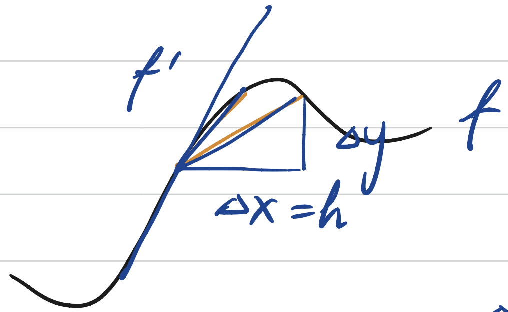
$f'(x) \qquad \frac{d}{dx} f(x)$

def.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\leftarrow \Delta y$

$\leftarrow \Delta x$



$$\frac{\Delta y}{\Delta x}$$

$\Delta x \to 0$

Derivative Laws

$$(f \pm g)' = f' \pm g'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain rule

$$(f \circ g)'$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} f(g(x)) \cdot \frac{d}{dx} g(x)$$

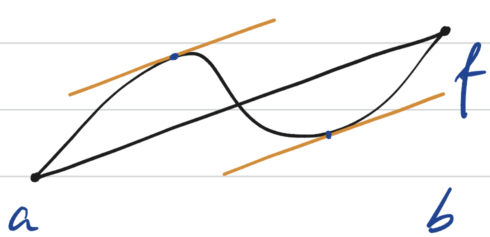
Derivatives of the following functions

f	x^n	e^x	$\sin x$	$\cos x$	$\ln x$
f'	nx^{n-1}	e^x	$\cos x$	$-\sin x$	$\frac{1}{x}$

$$\tan x' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos x \cdot \cos x - (-\sin x \cdot \sin x)}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Mean value theorem



Cont. $[a.b]$

there exists $c \in [a.b]$

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

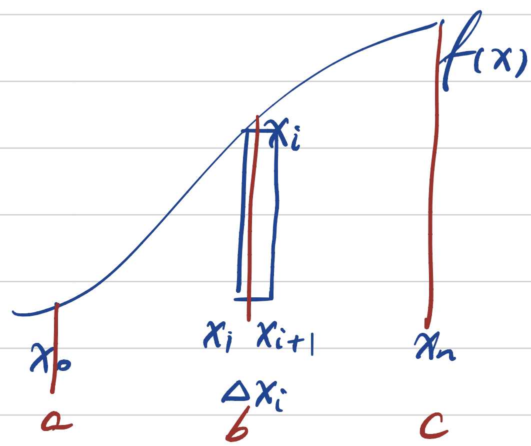
Integration

$\int dx$ "=" reversing $\frac{d}{dx}$.

limit of summation over small intervals.

$$\int \qquad \lim_{\Delta x_i \to 0} \sum_{i=0}^{n} f(x_i) \cdot \Delta x_i$$

Integration Laws.



lim

$$\int f+g \, dx = \int f \, dx + \int g \, dx$$

$$\int cf \, dx = c\int f \, dx$$

$$\int_a^b + \int_b^c = \int_a^c$$

Fundamental theorem of calculus. (FTC)

1. Given $F(x) = \int_a^x f(x) \, dx$

f cont. over $[a,b]$

$x \in [a,b]$

Then $F' = f$

2. $\int_a^b f(x) \, dx = F(b) - F(a)$ if f cont. over $[a,b]$, F antiderivative of f

! This allows us to compute integrals.

$$\int_0^1 x \, dx = \frac{1}{2}x^2\Big]_0^1 = \frac{1}{2}(1)^2 - \frac{1}{2}0^2$$

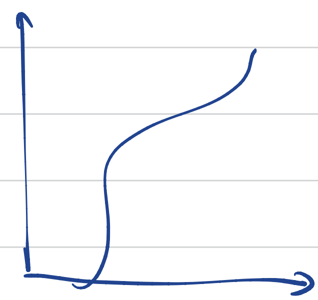
How to use Part 1 ?

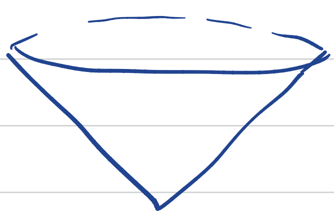
Example $\frac{d}{dx}\int_a^{\sin x} t^3 \, dt$

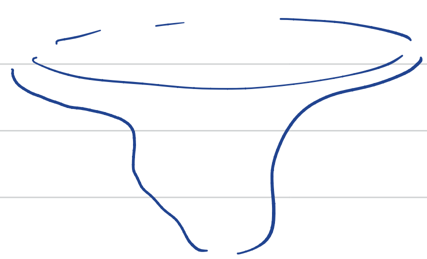
Take $u(x) = \sin x$ $\quad F(u) = \int_a^u t^3 \, dt$

$$\frac{d}{dx} F(u(x)) = \frac{dF}{du} \cdot \frac{du}{dx}$$
$$= u^3(x) \cdot \cos x$$
$$= \sin^3 x \cdot \cos x$$

Application: Computing area







Review Calc I (Substitution rule)

$$\int f(u(x))\, u'(x)\, dx = \int f(u)\, du.$$

Recall chain rule

$$(F \circ u)'(x) = F'(u(x)) \cdot u'(x)$$

Let f be the antiderivative of F.

$$\Rightarrow (F \circ u)'(x) = f(u(x)) \cdot u'(x)$$

Integrating w.r.t. x.

$$\int_a^b (F \circ u)'(x)\, dx = \int_a^b f(u(x)) \cdot u'(x)\, dx$$

FTC

$$= (F \circ u)(b) - (F \circ u)(a)$$

$$= F(u(b)) - F(u(a))$$

FTC

$$= \int_{u(a)}^{u(b)} f(u)\, du$$