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Week 1 Wed — Calc I Review & §7.1

This box contains useful tips for the question.

- When performing substitution, ensure that you adjust the limits of integration accordingly. E.g. $x \in [1, 2]$, $u = 2x$ belongs to $[2, 4]$.
- For indefinite integrals, remember to include the constant C in your final answers.

1. Evaluate the following integrals using the **substitution rule**. For each integral, specify u, $\mathrm{d}u$.

   $$\int_{\pi^2/36}^{\pi^2/16} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, \mathrm{d}x$$

2. Evaluate the following integrals using **Integration by Parts** (IBP). Substitution may also be necessary. For each integral, specify the functions $u, v,\ \mathrm{d}u$ and $\mathrm{d}v$.

   (a) $\displaystyle\int \frac{2\ln x}{x^3}\, \mathrm{d}x$

(b) $\displaystyle\int \arctan\sqrt{x}\,\mathrm{d}x$

(c) $\displaystyle\int e^{x}\cos x\,\mathrm{d}x$

WEEK 1 FRI — §7.2

- Write down the trigonometric identity and the double- and half-angle formulae.
- For indefinite integrals, remember to include the constant C in your final answers.

Evaluate the following **trigonometric integrals**.

1. $\displaystyle\int \sin^2 x \cos^2 x \, \mathrm{d}x$

2. $\displaystyle\int \sin^2 x \cos^5 x \, \mathrm{d}x$

Week 2 Wed — §7.2 & §7.3

- For indefinite integrals, remember to include the constant C in your final answers.

1. Fill in the table below.

	x	Range of θ	$\mathrm{d}x$	$\sqrt{\cdots}$ becomes
$\sqrt{a^2 - x^2}$			$a\cos(\theta)\ \mathrm{d}\theta$	$a\cos(\theta)$
$\sqrt{x^2 + a^2}$	$a\tan(\theta)$		$a\sec^2(\theta)\ \mathrm{d}\theta$	
$\sqrt{x^2 - a^2}$	$a\sec(\theta)$			$a\tan(\theta)$

2. In this question, we compute $I = \int \sec^3 x\ \mathrm{d}x$. The result will be used in the next question.

   (a) Start with integration by parts, applied to $I = \int \sec x \sec^2 x\ \mathrm{d}x$.

   (b) Use the trig identity $\tan^2 + 1 = \sec^2 x$. to break the $\int v\ \mathrm{d}u$ term into two parts.

(c) Solve for I. You may use $\displaystyle\int \sec x \, \mathrm{d}x = \ln|\sec x + \tan x| + C$ without justification.

(If you have spare time, you can return to this part and read the computation of $\displaystyle\int \sec x \, \mathrm{d}x$ given here.)

- It is important to specify the range of θ when applying the trig substitution.

3. $\displaystyle\int \sqrt{x^2 + 16} \, \mathrm{d}x$

After applying the trig substitution, you will need the integral we computed above

$$\int \sec^3 \theta \, \mathrm{d}\theta = \frac{1}{2} \sec\theta \tan\theta + \frac{1}{2} \ln|\sec\theta + \tan\theta| + C.$$

Week 2 Fri — §7.4

- Question 2 is postponed to the next discussion.
- For the upcoming quiz, the section covering §7.4 will focus solely on partial fraction decomposition.

1. Write out the form of the partial fraction decomposition for the following rational functions. **Do not solve** for the constants; leave them as A, B, C etc.

   (a) $\dfrac{x^3 + 2x + 5}{(x^2 + 4)(x - 1)(x - 3)}$

   (b) $\dfrac{x^6 + 5x^5 + 3x + 2}{(x^2 + 1)^2(x^2 + x + 1)x}$

Week 3 Wed — §7.4 & §7.7

- You may want to leave the questions marked with $\star$ for last, as they are more challenging.

1. Use partial fraction decomposition to evaluate the following integrals.

   (a) $\displaystyle\int \frac{x^2}{x-3}\,\mathrm{d}x$

   (b) $\displaystyle\int \frac{3x+11}{(x-3)(x+2)}\,\mathrm{d}x$

(c) ⋆ $\displaystyle\int \frac{dx}{x+\sqrt[3]{x}}$ (Hint: Try applying a substitution rule to get rid of $\sqrt[3]{x}$.)

Midpoint rule

$$\int_a^b f(x)\,\mathrm{d}x \approx M_n = (f(\overline{x_1}) + f(\overline{x_2}) + \cdots + f(\overline{x_n}))\,\Delta x.$$

where $\overline{x_i}$ are the midpoints and Δx is the width of each subinterval.

2. For the midpoint rule with $n = 6$ rectangles over the interval $[1, 7]$, calculate Δx and list the midpoints.

   $\Delta x =$__________________.
   The midpints are __________________.

3. Use the Midpoint Rule with $n = 6$ to approximate $\displaystyle\int_1^7 \frac{1}{x^3+1}\,\mathrm{d}x$.

Week 4 Wed — Midterm 1 review

Exam Info

- The exam on Friday covers §7.1–§7.4 and §7.7. Make sure that you can solve:
  - examples from lecture notes and the review session,
  - questions from quizzes, homework, and discussion worksheets.
- You are expected to know:
  - formulae for substitution, IBP,, trigonometric identities, and double-/half-angle formulae,
  - long division and partial fraction decomposition.
- The exam may include multiple-choice, true/false, fill-in-the-blank, and computational questions, similar to those found in quizzes, worksheets, and homework.
- The exam is **45 minutes** long and will be held **during the usual class time**. It is about 4 times the length of a quiz.
- If you have sent me a DRES letter previously requesting extended exam time, please make sure to request a location with the DRES office.

You may want to skip some questions during the class, as this worksheet is too lengthy to complete within 45 minutes.

Comments and suggestions from the review session

- You will **not** be asked to do long division on Midterm 1, so feel free to skip Q4.
- There aren't any questions of **trig integrals involving $\tan x$ and $\sec x$**. However, please **review all such questions from your homework, lecture notes, and discussion worksheets**. I apologize if the review sheet was misleading, as I wrote the exam after the review worksheet.
- The expectation for Midterm 1 is that you should be able to solve problems similar to those you have seen before. You will **not** be expected to tackle the most difficult problems during the midterm, as the time is limited. However, such problems may appear on a longer exam, such as the final.

1. Evaluate the following integrals using integration by parts. The substitution rule may be required for some problems.

   (a) $I = \displaystyle\int e^x \cos x \, \mathrm{d}x$

   (b) $I = \displaystyle\int \frac{8 \ln x}{x^5} \, \mathrm{d}x$

Again, please review the **trig integrals involving** $\tan x$ **and** $\sec x$ from your homework, lecture notes, and discussion worksheets.

2. Trigonometric integration: Evaluate the following integrals of the form $\int \sin^n x \cos^m x \, \mathrm{d}x$.

   (a) $I = \int \sin^5 x \, \mathrm{d}x$

   (b) $I = \int 16 \sin^2 x \cos^4 x \, \mathrm{d}x$

3. Trigonometric substitution.

   (a) $\displaystyle\int \frac{1}{\sqrt{x^2+6x}}\,\mathrm{d}x$. Feel free to use $\displaystyle\int \sec\theta\,\mathrm{d}\theta = \ln|\sec\theta+\tan\theta| + C$.

   (b) $\displaystyle\int (x-1)\sqrt{3+2x-x^2}\,\mathrm{d}x$

Again, you will **not** be asked to do long division on Midterm 1, so feel free to skip Q4.

4. Here are some problems for practicing long division.

   (a) $\dfrac{x^2}{x+1}$

   (b) $\dfrac{6x^2-3x}{(x-2)(x+4)}$

   (c) $\dfrac{x^4-5x^3+6x^2-18}{x^3-3x^2}$

5. Evaluate the following integral.

(a) $\displaystyle\int \frac{4}{x^2 + 5x - 14}\,\mathrm{d}x$

(b) $\displaystyle\int \frac{x+3}{(x^2+1)(x-2)}\,\mathrm{d}x$

6.

Trapezoidal rule

$$\int_a^b f(x)\,\mathrm{d}x \approx T_n = \frac{\Delta x}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right].$$

Use the trapezoidal rule to estimate $\int_0^1 x^2\,\mathrm{d}x$ using four subintervals.

Week 5 Mon — §7.8

- Review limit techniques from Calculus I and the steps for improper integrals and the comparison test in lecture notes.
- Read the remaining examples in §7.8 of the lecture notes.

1. Determine whether each of the given integrals is proper or improper. If an integral is improper, specify the point or points that make it improper. You **do not need to compute** the integrals or analyze their convergence or divergence.

   (a) $\displaystyle\int_{-\frac{1}{2}}^{7} \frac{1}{1-x^3}\,\mathrm{d}x$

   (b) $\displaystyle\int_{0}^{\infty} \frac{2+\cos(x)}{x^2-9x+8}\,\mathrm{d}x$

2. Use the **definition** of improper integrals to determine whether each given integral converges. If the integral converges, compute its exact value.

   (a) $\displaystyle\int_{1}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x$

   (b) $\displaystyle\int_{1}^{e} \frac{1}{x\ln x}\,\mathrm{d}x$

3. Use the Comparison Theorem to determine whether the given improper integral converges.

   (a) $\displaystyle\int_0^1 \frac{e^x}{\sqrt{x}}\,\mathrm{d}x$

   (b) $\displaystyle\int_1^\infty \frac{\cos^2(x)}{x^3+e^x}\,\mathrm{d}x$

Week 5 Wed — §7.8

1. For $y = x^3$ with $x \in [1, 2]$, **write down, but do not evaluate**, an integral with respect to x representing the arc length. Then, write the integrals with respect to y.

2. Compute the arc length of the following curves

   (a) $y = \cosh x$ for $x \in [0, \ln 2]$.

   (Recall: The hyperbolic cosine function $\cosh x$ is given by $\cosh x = \dfrac{e^x + e^{-x}}{2}$).

   (b) $y = \dfrac{2}{3}(1 + x^2)^{3/2}$ starting from the point $(0, \dfrac{2}{3})$.

3. Find the arc length of the portion of the implicitly defined curve $x^{2/3} + y^{2/3} = 1$ that lies in the first quadrant.
(Hint: the curve is bounded between $(0, 1)$ and $(1, 0)$. Feel free to ask if you need a reminder on how to perform implicit differentiation.)

Week 5 Fri — §8.1 & §8.2

1. For the following curve, which is rotated about a line in the xy-plane, **set up, but do not evaluate**, an integral that represents the surface area of the resulting solid.

   (a) The curve $y = \arctan x$, for $0 \leq x \leq 1$ rotated about the y-axis.

   (b) The curve $y = x^2$, for $0 \leq x \leq 1$ rotated about the line $y = -1$.

2. Consider the curve $y = x^3$ between $(0, 0)$ and $(1, 1)$, rotated about the x-axis.

   Set up and **evaluate** an integral with respect to x that represents the area of the resulting surface.

3. If you have extra time, challenge yourself to derive the *arc length* and *surface area* formula on your own without consulting your notes. This exercise will strengthen your comprehension and serve as a helpful backup if you forget the formula during an exam.

Week 6 Wed — §10.1 & §10.2

1. Consider the parametric curve defined by $x(t) = t^2$ and $y(t) = t^3 - 3t$, for $-\infty < t < \infty$.

    (a) Find all the x-intercepts and y-intercepts and the corresponding t-value. **Note:** x-intercepts occur when $y = 0$, and y-intercepts occur when $x = 0$.

    (b) Determine the intervals where $x(t)$ and $y(t)$ are increasing or decreasing. Find the t-value corresponding to a critical point (when the derivative is zero).

    (c) Find the x, y-coordinates of points where the curve has **horizontal tangents** ($\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$) or **vertical tangents** ($\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$).

2. Consider the curve parametrized by

$$x(t) = 2\cos(t) \quad , y(t) = 3\sin(t), \quad 0 \le t \le 4\pi.$$

(a) **Set up (do not evaluate)** the arc length of this curve.

(b) Describe the geometric shape of the curve and explain how the parameter interval $0 \le t \le 4\pi$ affects the tracing of this shape.

3. Compute the surface area of the solid obtained by rotating the following parametric curve about the x-axis.

$$x = \cos^3\theta, \quad y = \sin^3\theta, \quad 0 \le \theta \le \frac{\pi}{2}.$$

Week 6 Fri — §10.2 & §10.4

1. Coordinate conversion.
   (a) Convert the polar coordinates $(r, \theta) = (\sqrt{2}, \frac{\pi}{4})$ to rectangular coordinates.

   (b) Convert the rectangular coordinates $(x, y) = (-1, -\sqrt{3})$ to polar coordinates.

   (c) $\star$ Convert the rectangular coordinates $(x, y) = (1, -1)$ to polar coordinates. Give an answer with $r < 0$.

2. Sketch the regions.
   (a) $1 \leq r \leq 2, \quad -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{4}$.
   (b) $r \leq 0, \quad \frac{4}{3}\pi \leq \theta \leq \frac{5}{3}\pi$.

We didn't cover the rest of the questions (Q3-5) on Friday. However, since we won't have enough time for a discussion next Monday, I'll provide the solution. Please try to solve the problems when you have time.

3. Find the tangent line to $r = \sin(4\theta)\cos(\theta)$ at $\theta = \dfrac{\pi}{6}$.

4. A four-leaved rose is represented in polar coordinates by the equation $r = \sin(2\theta)$. Find the area of the region enclosed by a single loop.

5. Find the arc length of the polar curve $r = 2\sin\theta$, over the interval $0 \le \theta \le \dfrac{\pi}{3}$.

Week 7 Wed — Midterm 2 review

Exam Info

- The exam on Friday covers §7.8, §8.1-§8.2 and §10.1-§10.3. Make sure that you can solve:
  - examples from lecture notes and the review session,
  - questions from quizzes, homework, and discussion worksheets.
- You are expected to know:
  - The definitions of proper and improper integrals, methods for solving them, and the comparison test for improper integrals.
  - The infinitesimal line element and area element, and their use in computing arc length and the surface area of revolution for curves defined in both rectangular coordinates and parametrization.
  - Converting between rectangular and polar coordinates, as well as how to graph polar curves.
- The exam may include multiple-choice, true/false, fill-in-the-blank, and computational questions, similar to those found in quizzes, worksheets, and homework.
- The exam is **45 minutes** long and will be held **during the usual class time**. It is about 4 times the length of a quiz.
- If you have sent me a DRES letter previously requesting extended exam time, please make sure to request a location with the DRES office.

You may want to skip some questions during the class, as this worksheet is too lengthy to complete within 45 minutes.

Comments and suggestions from the review session

- You will **not** be asked to do long division on Midterm 1, so feel free to skip Q4.
- There aren't any questions of **trig integrals involving** $\tan x$ **and** $\sec x$. However, please **review all such questions from your homework, lecture notes, and discussion worksheets**. I apologize if the review sheet was misleading, as I wrote the exam after the review worksheet.
- The expectation for Midterm 1 is that you should be able to solve problems similar to those you have seen before. You will **not** be expected to tackle the most difficult problems during the midterm, as the time is limited. However, such problems may appear on a longer exam, such as the final.

1. **True or False:**
   (a) __________ An improper integral is defined using limits.
   (b) __________ $\int_{-2}^{3} \frac{1}{x}\, \mathrm{d}x$ is an improper integral.
   (c) __________ Since $\int_{0}^{\infty} e^{-x}\, \mathrm{d}x$ is an improper integral, it diverges.

2. Consider two improper integrals $\int_a^b f(x)\, \mathrm{d}x$ and $\int_a^b g(x)\, \mathrm{d}x$. Assume $0 \leq f(x) \leq g(x)$ for all $a < x < b$. Choose **two** correct statements from the Comparison Theorem.
   A. If $\int_a^b g(x)\, \mathrm{d}x$ converges, then $\int_a^b f(x)\, \mathrm{d}x$ also converges.
   B. If $\int_a^b g(x)\, \mathrm{d}x$ diverges, then $\int_a^b f(x)\, \mathrm{d}x$ diverges.
   C. If $\int_a^b f(x)\, \mathrm{d}x$ converges, then $\int_a^b g(x)\, \mathrm{d}x$ converges.
   D. If $\int_a^b f(x)\, \mathrm{d}x$ diverges, then $\int_a^b g(x)\, \mathrm{d}x$ diverges.

3. Evaluate the improper integral $\int_e^{\infty} \frac{1}{x(\ln x)^2}\, \mathrm{d}x$ or show that it diverges.
   (Hint: Use the substitution $u = \ln x$.)

4. Consider the curve $y = e^x$ between the points $(0, 1)$ and $(2, e^2)$. Let S be the surface obtained by revolving this curve around the y-axis.

   (a) Express $\mathrm{d}s$ in terms of $\mathrm{d}x$.

   (b) Express $\mathrm{d}s$ in terms of $\mathrm{d}y$.

   (c) Find the correct integral that represents the arc length of the curve.

   A. $\displaystyle\int_0^1 \sqrt{1+e^x}\,\mathrm{d}x$ B. $\displaystyle\int_0^2 \sqrt{1+e^{2x}}\,\mathrm{d}x$ C. $\displaystyle\int_1^{e^2} \sqrt{1+e^{2x}}\,\mathrm{d}y$ D. $\displaystyle\int_1^{e^2} \sqrt{1+y^2}\,\mathrm{d}y$

   (d) Recall that the surface area $A = \displaystyle\int 2\pi R\,\mathrm{d}s$. In this case, $R =$?

   ○ $R = x$ ○ $R = y$

   (e) Set up, but do not evaluate, an integral with respect to x that represents the surface area of S. Then do the same for y

5. Consider the parametric curve given by:

$$(x, y) = (1 + 2t,\, 2 + 4t), \quad -\infty < t < \infty.$$

   (a) Eliminate the parameter t to find an equation involving only x and y.

(b) Identify the curve represented by the parametric equations.

6. Write parametric equations for a circle centered at the origin with radius 3. Include the appropriate bounds for the parameter.

7. Consider the parametric curve:

$$(x, y) = \left(t^2 - 2t,\ t^3 - 6t\right), \quad -\infty < t < \infty.$$

(a) Determine the points on the curve where the tangent line has a slope of 3.

(b) Write the equations of the tangent lines at these points.

8. Match the formula with the quantity it computes.

A. $\displaystyle\int_a^b \sqrt{f'(t)^2 + g'(t)^2}\,\mathrm{d}t$

B. $\displaystyle\left|\int_a^b g(t)f'(t)\,\mathrm{d}t\right|$

C. $\displaystyle\int_a^b 2\pi f(t)\sqrt{f'(t)^2 + g'(t)^2}\,\mathrm{d}t$

(a) ________The arc length of the curve parametrized by $(x, y) = (f(t), g(t)), a \le t \le b$.

(b) ________The area between the x-axis and the curve parametrized by $(x, y) = (f(t), g(t)), a \le t \le b$.

(c) ________The area of the surface obtained by rotating the curve parametrized by $(x, y) = (f(t), g(t)), a \le t \le b$ around the y-axis.

9. Convert the polar equation $r = \cos\theta + \sin\theta$ to an equation in terms of only x, y.

10. Match each polar equation with the curve it describes.

A. $r = 2$

B. $r = 2\cos\theta$

C. $\theta = 1$

D. $r = 2\sin\theta$

E. $r = 2\csc\theta$

F. $r = \sec\theta$

(a) ________A straight line that is neither vertical nor horizontal.

(b) ________A circle whose center is $(1, 0)$.

(c) ________A horizontal line.

(d) ________A circle whose center is $(0, 0)$.

(e) ________A vertical line.

(f) ________A circle whose center is $(0, 1)$.

11. Consider the parametric curve $x = \sin^2 t, y = \sin(3t)$, $0 \le t \le \dfrac{\pi}{3}$. Set up, but do not evaluate, integrals which represent the following:
   (a) The area under the curve.

   (b) The surface area created by rotating the curve about the x-axis.

12. Graph the following polar curves
   (a) $r \cos\theta = 6$
   (b) $\theta = \dfrac{\pi}{6}$
   (c) $r = 7$
   (d) $r = 4\cos\theta$
   (e) $r = -7\sin\theta$
   (f) $r = 2 + 4\cos\theta$
   (g) $r = 4 - 9\sin\theta$
   (h) $r = \sin(2\theta)$
   (i) $r = \cos(3\theta)$

Week 8 Wed — §11.1 & 11.2

1. For each of the following sequences, determine a formula for the general term a_n (assuming n starts from 1) and compute $\lim\limits_{n\to\infty} a_n$.

   (a) $\left\{\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \cdots\right\}$

   (b) $\left\{-\frac{2}{3}, \frac{4}{9}, -\frac{6}{27}, \frac{8}{81}, \cdots\right\}$

   (c) $\left\{(1+1)^1, (1+\frac{1}{2})^2, (1+\frac{1}{3})^3, (1+\frac{1}{4})^4, \cdots\right\}$

   (d) $\left\{\frac{7}{7}, -\frac{9}{11}, \frac{11}{15}, -\frac{13}{19}, \cdots\right\}$

   (e) $\left\{\frac{\cos(2)}{1}, \frac{\cos(4)}{2}, \frac{\cos(6)}{3}, \frac{\cos(8)}{4}, \frac{\cos(10)}{5}, \frac{\cos(12)}{6}, \cdots\right\}$

2. For each of the following sequences, determine whether it is monotonic and whether it is bounded.

   (a) $a_n = n$

   (b) $a_n = (-1)^n n$

   (c) $a_n = \frac{1}{n}$

3. In each part, determine whether the given facts are sufficient to conclude that the sequence converges, diverges, or if more information is needed.

   (a) Given that $2 < a_n < 31$ for all n, and that the sequence is increasing: $a_1 < a_2 < a_3 < a_4 < \cdots$. Then

   A. $\{a_n\}$ converges. B. $\{a_n\}$ diverges. C. More information is needed.

(b) Given that $a_n < 231$ for all n, but no lower bound exists for a_n. We can conclude that
A. $\{a_n\}$ converges. B. $\{a_n\}$ diverges. C. More information is needed.

4. Let $\{a_n\}$ be a sequence, and let s_N denote the partial sum of the series $\sum_{n=1}^{\infty} a_n$. Which of the following statements are correct?

A. $s_N = a_1 + a_2 + \cdots + a_N$ B. $s_N = s_{N+1} + a_{n+1}$ C. $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} s_N$

D. $a_N = s_1 + s_2 + \cdots + s_N$ E. $s_{N+1} = s_N + a_{N+1}$ F. $\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} a_N$

5. Find the sum of the following geometric series, or state 'Diverges' if the series does not converge.

(a) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \ldots$

(b) $\sum_{n=1}^{\infty} 5 \cdot (-2)^{n-1}$

(c) $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$

6. Prove that each of the following series diverges.

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

(b) $\sum_{n=1}^{\infty} e^{-\frac{n}{n^2+1}}$. (It's e raised to the power $-\frac{n}{n^2+1}$).

Week 8 Fri — §11.3

1. Consider the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

   (a) Use the Integral Test to show that the series diverges.

   (b) Let $S_N = \sum_{n=1}^{N} \frac{1}{n}$ be the N-th partial sum. Use a graph to illustrate that $S_N \leq 1 + \ln N$ by interpreting the sum as a right Riemann sum for an appropriate integral. (Hint: Consider the integral approximation using the function $f(x) = \frac{1}{x}$.)

2. Consider the series $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n(\ln n)^5}$.

   (a) Use the Integral Test to show that the series converges.

   (b) We want to estimate the value of the infinite series using S_N. Determine the minimum number of terms required to ensure that the error in the estimation, given by the remainder is at most 10^{-4}. (Hint: Use the Integral Test to bound $R_N = \displaystyle\sum_{n=2}^{\infty} \frac{4}{n(\ln n)^5} - S_N$.)

Week 10 Wed — §11.4 & §11.5

1. In this question, we consider the series $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sqrt[3]{8n^2-5}}{n^3+6}$.

   (a) Choose the series that should be used to apply **the Direct Comparison Test**:

   A. The p-series: $\displaystyle\sum_{n=1}^{\infty} \frac{a}{n^p}$. B. The geometric series: $\displaystyle\sum_{n=1}^{\infty} ar^{n-1}$.

   (b) Write down the corresponding inequality:

   (c) Based on the inequality above and **the Direct Comparison Test**, the series $\displaystyle\sum_{n=1}^{\infty} a_n$

   A. converges. B. diverges.

2. For each series below, construct an inequality for the Direct Comparison Test that allows you to determine whether or not the series converges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n\sin^2(n)}{n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)+5^n}{7^n}$

3. For each part, determine the corresponding b_n so that **the Limit Comparison Test** applies.

   Recall that you need $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c$ for some constant $0 < c < \infty$.

   (a) $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n^2-2n+1}{n^4+27n^3+1}$, $b_n =$ ______________.

   (b) $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sqrt{n^{16}+2\sqrt{n}+1}}{n^5+7\sqrt[5]{n}+1}$, $b_n =$ ______________.

   (c) $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1+2^n}{100+3^n}$, $b_n =$ ______________.

4. Use **the Alternating Series Test** to determine whether the series converges or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{(\ln n)^2}{n}$

   (b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{2n+5}{4n+7}$

5. Consider an alternating series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}b_n = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$.

   (a) Suppose the alternating series converges. Find the smallest integer N such that the error in the estimation is no more than $\dfrac{1}{100000}$ by the Alternating Series Estimation Theorem.

   (b) Find an upper bound for $|R_{3455}|$.

Week 11 Wed — Midterm 3 Review

Exam Info

- The exam on Friday covers §10.4 (bonus question) and §11.1–§11.5 (mandatory part). Make sure that you can solve:
  - examples from lecture notes and the review session,
  - questions from quizzes, homework, and discussion worksheets.
- You are expected to know:
  - How to compute the area bounded by polar curves, arc length, and surface area of revolution.
  - Definition of sequence, series, partial sum, and alternating series.
  - Harmonic series, geometric series, their convergence, and the value they converge to.
  - The divergence test, the integral test, the direct comparison test, the limit comparison test, and the alternating series test. (You are expected to know the statement of the theorems.)
  - How to **formulate a rigorous proof** using the convergence tests.
  - The statement of the error estimate and how to use the error estimate inequalities.
- The exam may include multiple-choice, true/false, fill-in-the-blank, and computational questions, similar to those found in quizzes, worksheets, and homework.
- The exam is **45 minutes** long and will be held **during the usual class time**. It is about 4 times the length of a quiz.
- If you have sent me a DRES letter previously requesting extended exam time, please make sure to request a location with the DRES office.

You may want to skip some questions during the class, as this worksheet is too lengthy to complete within 45 minutes.

1. Let $\{a_n\}$ be a sequence, s_N be the partial sum of the series $\sum_{n=1}^{\infty} a_n$, and R_N be the remainder. Suppose $\sum_{n=1}^{\infty} a_n$ converges. Which of the following are true? A. $R_N = a_1 + a_2 + \cdots + a_N$.
   B. $R_N = a_{N+1} + a_{N+2} + \ldots$.
   C. $R_N + s_N = \sum_{n=1}^{\infty} a_n$.
   D. $s_N = a_1 + a_2 + \cdots + a_N$.
   E. $s_N = a_{N+1} + a_{N+2} + \ldots$.
   F. $R_N = \sum_{n=1}^{\infty} a_n - s_N$.

2. To apply the integral test to the series $\sum_{n=1} \frac{\sin^2(n)}{n}$, start by defining the function $f(x) =$ __________. Next, verify that for some N, $f(x)$ is __________________, __________________, and __________________ for $x \geq N$.

3. In each part, decide whether you can apply the integral test or the alternating series test to the given series.
   (a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^5}$
   A. The Integral Test. B. The Alternating Series test. C. Neither could be applied.

   (b) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^5 + 4n + 9}$
   A. The Integral Test. B. The Alternating Series test. C. Neither could be applied.

   (c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{2n + 7}$
   A. The Integral Test. B. The Alternating Series test. C. Neither could be applied.

4. Consider a positive series $\sum_{n=1}^{\infty} a_n$. For each statement, determine if the series converges, diverges, or if the information is insufficient to draw conclusions.

   (a) $\lim_{n\to\infty} \frac{a_n}{2^n} = 231$

   A. The series converges. B. The series diverges. C. Insufficient information.

   (b) $\lim_{n\to\infty} \frac{a_n}{1/n^2} = 0$

   A. The series converges. B. The series diverges. C. Insufficient information.

   (c) $\lim_{n\to\infty} \frac{a_n}{7^n} = \infty$

   A. The series converges. B. The series diverges. C. Insufficient information.

5. For each part, either provide an example that satisfies the description or explain why such an example does not exist.

   (a) Give an example of a sequence a_n that is bounded below but not bounded above.

   (b) Give an example of a sequence a_n that is bounded but not monotonic.

   (c) Give an example of a sequence a_n that is both bounded and decreasing, but not convergent.

(d) Give an example of a sequence a_n such that $\lim\limits_{n\to\infty} |a_n| = 0$, but $\lim\limits_{n\to\infty} a_n \neq 0$.

(e) Give an example of a sequence a_n such that $(1+\frac{1}{n})^n \leq a_n \leq e$ and $\lim\limits_{n\to\infty} a_n$ does not exist.

(f) Give an example of a convergent series $\sum\limits_{n=1}^{\infty} a_n$ such that $\lim\limits_{n\to\infty} a_n = 1$.

(g) Give an example of a divergent series $\sum\limits_{n=1}^{\infty} a_n$ such that $\lim\limits_{n\to\infty} a_n$ exists.

Warning:

- For short answer questions, you always need to **check that the assumptions in the theorem hold** before you apply the theorem.
- Be careful with the number $\boldsymbol{N}$; it **may not match** the starting point of the series.

6. Decide whether each of the following series converges. If the series is a convergent geometric series, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} e^{\frac{2n-1}{4n+5}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{3}{2}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

7. (a) Use the Integral Test to decide whether $\sum_{n=2}^{\infty} \frac{1}{(2n-7)^3}$ converges or diverges.

(b) Write down the error bound for the integral test

(c) If we want to estimate the series with an upper error bound no greater than $\frac{1}{10000}$, what is the **smallest** N that we should take? You may use the fact that

$$\int_a^{\infty} \frac{1}{(2x-7)^3}\,\mathrm{d}x = \frac{1}{4}\frac{1}{(2a-7)^2}.$$

8. Use the Comparison Test to decide whether $\displaystyle\sum_{n=0}^{\infty} \frac{2^n \sin^2(5n)}{4^n + \cos^2(n)}$ converges or diverges.

9. Use the Limit Comparison Test to decide whether $\displaystyle\sum_{n=2}^{\infty} \frac{4n^2 + n}{\sqrt[3]{n^7 + n^3}}$ converges or diverges.

10. (a) Use the Alternating Series Test to decide whether $\sum_{n=0}^{\infty} \frac{(-1)^{n-2}\sqrt{n}}{n+4}$ converges or diverges.

(b) Find the error bound of $\sum_{n=0}^{5} \frac{(-1)^{n-2}\sqrt{n}}{n+4}$ for the infinite series.

Week 12 Wed — §11.6 & §11.8

This worksheet is here to give you extra examples with complete solutions for §11.6 and §11.8. It's a bit long to cover in class, so please choose one part from each question to work through. If you get stuck, feel free to ask questions.

1. Determine if the following series converge absolutely, converge conditionally, or diverge.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

2. Determine the **radius of convergence** R and the **interval of convergence** I for the power series.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{(x-3)^n}{n \cdot 5^n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^{2n}}{(2n)!}$

Week 13 Wed — §11.9

1. Consider the function $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$.

   (a) Verify the identity
   $$\ln\left(\frac{1+x}{1-x}\right) = \int \left(\frac{1}{1+x} + \frac{1}{1-x}\right)\, \mathrm{d}x + C.$$
   Hint: Use property of ln *and the Fundamental Theorem of Calculus.*

   (b) Using part (a), write a power series for $f(x)$ using term-by-term integration.
   Hint: Recall the geometric series formulas: $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, *valid for* $|x| < 1$.

   (c) Use an initial condition (such as $x = 0$) to determine the constant C and give the final answer.

2. Consider the rational function $f(x) = \dfrac{6x}{3x^2 + 2x - 1}$.

   (a) Decompose $f(x)$ into a sum of simpler rational functions.
   Hint: Factor the denominator and find constants A, B such that
$$\frac{6x}{(x+1)(3x-1)} = \frac{A}{x+1} + \frac{B}{3x-1}.$$

   (b) Expand the expression obtained above as a power series.

   (c) Determine the interval of converge, and express the final result as $\displaystyle\sum_{n=0}^{\infty} c_n x^n$.
   Hint: Since the final series is formed by combining two separate series, both must converge for the result to be valid.

Week 14 Mon — §11.10

1. Use the definition to find the Taylor series of $f(x) = e^x$ **centered at $a = 1$** and determine its radius of convergence.

2. Find the **2nd degree Taylor polynomial** $T_2(x)$ of $f(x) = x^{3/2}$, centered at $a = 4$.

3. You are given that the Maclaurin series of $\sin(x)$ is $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, -\infty < x < \infty$.

(a) Use this series to find the Maclaurin series of $\cos x$ by **differentiating** both sides.

(b) Use the result from part (a) to find the Maclaurin series of $f(x) = x^3 \cos(x^2)$.

(c) Use the result from part (b) and the definition of the Maclaurin series to find $f^{(3)}(0), f^{(5)}(0), f^{(7)}(0)$.

Week 14 Wed — §11.11

In this worksheet, we want to approximate $(4.5)^{\frac{3}{2}}$ by Taylor polynomials.

1. What's the 2nd-order Taylor polynomial $T_2(x)$ for $f(x) = x^{\frac{3}{2}}$ centered at 4.

2. Use $T_2(x)$ in question (a) to approximate $(4.5)^{\frac{3}{2}}$. Leave your answer as a sum of fractions.

3. Find the maximal value of $|f^{(3)}(x)|$ on the interval $[4, 4.5]$.

4. Find an upper bound of $|(4.5)^{\frac{3}{2}} - T_2(4.5)|$ given by the Taylor's Inequality.

Week 14 Fri — §11.11

In this worksheet, we aim to estimate the integral $\int_0^1 \cos(\sqrt{x})\,\mathrm{d}x$ by expanding the integrand into a power series and computing an approximation with an error less than 0.01.

1. Expand $\cos(\sqrt{x})$ as a Maclaurin series. Recall that $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$.

2. Obtain a series expansion for the integral by integrating term-by-term.

3. Write down the error bound for the series you get from Q2.
   Hint: the Alternating Series Estimation

4. Use Q3 to determine the smallest integer N such that the approximation error $R_N < 0.01$.
   Hint: Because of the factorial, it is easier if you try to plug in $N = 0, 1, 2 \cdots$.

Week 15 Wed — Midterm 4 review

Exam Info

- The exam on Friday covers §11.6–§11.11. Make sure that you can solve:
  - examples from lecture notes and the review session,
  - questions from quizzes, homework, and discussion worksheets.
- You are expected to know:
  - The definitions of absolute convergence and conditional convergence.
  - The Ratio Test and the Root Test.
  - The radius and interval of convergence of a power series.
  - Taylor and Maclaurin series: deriving and applying them for approximations.
  - Estimating integrals or functions using Taylor series.
  - Using Taylor series to compute limits (bonus question).
  - How to **formulate a rigorous proof** using convergence tests.
  - The statement of the error estimate (Taylor's inequality) and how to apply it.
- Even though it is not listed above, when checking the endpoints of the interval of convergence, you will need knowledge of:
  - The harmonic and geometric series, their convergence properties, and the values to which they converge.
  - The Divergence Test, the Integral Test, the Direct Comparison Test, the Limit Comparison Test, and the Alternating Series Test.
- The exam may include multiple-choice, true/false, fill-in-the-blank, and computational questions, similar to those found in quizzes, worksheets, and homework.
- The exam is **45 minutes** long and will be held **during the usual class time**. It is about 4 times the length of a quiz.
- If you have sent me a DRES letter previously requesting extended exam time, please make sure to request a location with the DRES office.

You may want to skip some questions during the class, as this worksheet is too lengthy to complete within 45 minutes.

1. Determine if the following series converge absolutely, converge conditionally, or diverge. Justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{(b_n)^n}$, where $\{b_n\}$ is a positive sequence such that $\displaystyle\lim_{n\to\infty} b_n = 2$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{3n+4}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n+5)}{3+2^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{2^n}$

2. Consider the series $\sum_{n=2}^{\infty} \frac{3}{\ln n}$. What does **the Ratio Test** conclude?

   A. The Ratio Test is inconclusive.

   B. The Ratio Test shows that this series is absolutely convergent.

   C. The Ratio Test shows that this series is divergent.

3. For each of the following infinite series, determine whether **the Ratio Test** or **the Root Test** is more appropriate.

   (a) $\sum_{n=1}^{\infty} (-1)^n (\frac{\ln n}{4n})^n$ __________________________.

   (b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ __________________________.

   (c) $\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot 10 \cdots (3n+1)}{3 \cdot 5 \cdot 7 \cdot 9 \cdots (2n+3)}$ __________________________.

   (d) $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$ __________________________.

4. For each case below, decide if a power series centered at a can have the given interval of convergence I and radius of convergence R.

   (a) $I = (3, 11)$ and $R = 5$. A. possible B. not possible

   (b) $I = (-\infty, 1]$. A. possible B. not possible

   (c) $I = [-7, 9)$ and $a = 1$. A. possible B. not possible

   (d) $I = [-2, 8]$ and $R = 5$. A. possible B. not possible

5. A power series converges when $x = 4, 5, 8$, but diverges when $x = 0, 3, 10$. Which of the points below could be the center a of the series?
   (a) $a = 6$. A. Possible. B. Impossible.
   (b) $a = 7$. A. Possible. B. Impossible.
   (c) $a = 8$. A. Possible. B. Impossible.
   (d) $a = 9$. A. Possible. B. Impossible.

6. Consider the power series $\sum c_n(x-3)^n$.
   (a) What is the center of this series? ______________
   Suppose the radius of convergence is $R = 2$. For each value of x below, determine whether the resulting series converges, diverges, or if there is insufficient information.
   (b) $x = 0$, i.e. $\sum c_n(-3)^n$ A. Converges B. Diverges C. Cannot determine
   (c) $x = 2$, i.e. $\sum c_n(-1)^n$ A. Converges B. Diverges C. Cannot determine
   (d) $x = 5$, i.e. $\sum c_n(2)^n$ A. Converges B. Diverges C. Cannot determine

7. Consider the power series $\sum c_n x^n$. Given that $\sum c_n 4^n$ converges and $\sum c_n(-7)^n$ diverges, determine the behavior of each of the following series.
   (a) $\sum c_n(-9)^n$
   A. Converges B. Diverges C. Not enough information
   (b) $\sum c_n 2^n$
   A. Converges B. Diverges C. Not enough information
   (c) $\sum c_n 5^n$
   A. Converges B. Diverges C. Not enough information
   (d) $\sum c_n(-4)^n$
   A. Converges B. Diverges C. Not enough information

8. Find **the radius and interval of convergence** of the power series $\displaystyle\sum_{n=2}^{\infty} \frac{x^{4n}}{n(\ln n)^5}$.

In the exam, you will be given the following Maclaurin series.

$$\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad |x| < \infty.$$

$$\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}, \quad |x| < \infty.$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^{n}}{n}, \quad |x| < 1.$$

$$\arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}, \quad |x| < 1.$$

$$(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2}x^2 + \cdots, \quad |x| < 1$$

9. Use substitution, differentiation, and integration to find a power series representation for each of the following functions. **Be sure to include the radius of convergence.**

   (a) $f(x) = \ln(\frac{1+x}{1-x})$. Recall that $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$

   (b) $f(x) = \frac{1}{(1+x)^2}$.

10. (4 points) Given that $f(x)$ has a power series representation $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}, |x| < 1$.

(a) What's the power series representation of $f(2x)$?

(b) What's the power series representation of $f'(x)$?

(c) What's the power series representation of $\int f(x)\, dx$?

11. Use the Maclaurin series to evaluate the following series.

(a) $1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots$

(b) $1 - \frac{\pi^2}{2^2 2!} + \frac{\pi^4}{2^4 4!} - \frac{\pi^6}{2^6 6!} + \cdots$

(c) $-\frac{\pi^3}{2!} + \frac{\pi^5}{4!} - \frac{\pi^7}{6!} + \frac{\pi^9}{8!} - \cdots$

12. Use the Taylor series of ln to determine the behavior of the series $\displaystyle\sum_{n=1}^{\infty} -\frac{(-2)^n}{n}$.

A. The series converges and is equal to ln(3).

B. The series converges to a value close to, but not equal to, ln(3).

C. The series diverges.

D. None of the above.

13. True/False.

(a) ________ If the Maclaurin series of a function $f(x)$ has the radius of convergence 1, then the Maclaurin series of the derivative function $f'(x)$ also has the radius of convergence 1. In other words, taking derivative doesn't change the radius of convergence of the Maclaurin series.

(b) ________ The 2^{nd} degree Taylor polynomial $T_n(x)$ of the function $f(x) = e^x$ centered at 0 is $T_n(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$.

(c) ________ The n^{th} degree Taylor polynomial of a function $f(x)$ centered at a is the n^{th} partial sum of the Taylor series of $f(x)$ centered at a.

(d) ________ Suppose the Macaurin series of $f(x)$ is $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}x^n = 1 + 3x + \frac{9}{2}x^2 + ...$ Then $f''(0) = \frac{9}{2}$.

(e) ________ Suppose the radius of convergence of the Maclaurin series of a function $f(x)$ is 1. Then plugging $x = 2$ in the Macaurin series will give us $f(2)$.

(f) ________ The binomial coefficient $\binom{-\frac{1}{2}}{2}$ is

$$\frac{(-\frac{1}{2}) \cdot (-\frac{1}{2} - 1)}{2!} = \frac{3}{8}.$$

14. Suppose that $h(x)$ is a function that is continuous for all real values of x. Let's say that you want to compute the third degree Taylor polynomial $T_3(x)$ centered at $a = 3$ for $h(x)$, and you know that $h(3) = 1, h'(3) = -6, h''(3) = 4$ and $h'''(3) = 48$. What's $T_3(x)$?

15. Find the third degree Taylor polynomial $T_3(x)$ centered at $a = 1$ for the function $f(x) = x^4 + 5x^3 - 2x^2 + 5$.

16. Use multiplication to find the Maclaurin series of $e^x \arctan(x)$ up to order 4 and then evaluate the limit

$$\lim_{x \to 0} \frac{e^x \arctan(x) - x - x^2}{x \sin(x^2)}.$$

17. Suppose $f(x)$ has the Maclaurin series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{6^n}$.

   (a) Use this series and the definition to find $f^{(10)}(0), f^{(11)}(0)$.

   (b) Find a power series representation of $\displaystyle\int x \cdot f(4x^2)\,dx$ and find its the radius of convergence.

- Time and date: May 15, 1:30-4:30pm
- Location: 40 Allen Hall (our classroom)
- The exam is **3 hours** long, about four times the length of a midterm.
- If you have sent me a DRES letter previously requesting extended exam time, please make sure to request a location with the DRES office.
- The exam may include multiple-choice, true/false, fill-in-the-blank, and computational questions, similar to those found in quizzes, worksheets, and homework.
- Make sure that you can solve:
  - Examples from lecture notes and the review session,
  - Questions from quizzes, homework, and discussion worksheets.

  You will see similar questions.
- **Checklist of materials to know:**
  - **Chapter 7: Integration techniques**
    * Substitution rule
    * Integration by parts
    * Trigonometric identities
    * Improper integrals
    * Partial fraction decomposition
  - **Chapter 8: Applications of integration**
    * Arc length and surfaces of revolution
    * Surface area enclosed by a curve
  - **Chapter 10: Parametric equations and polar coordinates**
    * Parametrization of curves and surfaces
    * Polar curves
    * Arc length and surfaces of revolution
    * Surface area enclosed by a curve
  - **Chapter 11: Sequences and series**
    * Definitions of sequence, series, partial sum, and alternating series
    * Harmonic series, geometric series, and their convergence (including values they converge to)
    * Absolute convergence and conditional convergence
    * The divergence test, the integral test, the direct comparison test, the limit comparison test, and the alternating series test (including knowing the statement of the theorems)
    * The ratio test and the root test
    * Radius of convergence and interval of convergence of a power series
    * **Taylor and Maclaurin series**: Deriving and applying them for approximations
    * Estimating integrals or functions using power series
- **General knowledge (for all chapters):**
  - How to **formulate a rigorous proof** using the convergence tests.
  - The statement of the error estimate and how to use the error estimate inequalities.
  - **Differential equations**: Solving simple first-order differential equations using separation of variables.